In other words, the electromagnetic mass rises with velocity inversely as $\sqrt{1 - v^2/c^2}$—a discovery that was made before the theory of relativity.

Early experiments were proposed to measure the changes with velocity in the observed mass of a particle in order to determine how much of the mass was mechanical and how much was electrical. It was believed at the time that the electrical part *would* vary with velocity, whereas the mechanical part would *not*. But while the experiments were being done, the theorists were also at work. Soon the theory of relativity was developed, which proposed that no matter what the origin of the mass, it *all* should vary as $m_0/\sqrt{1 - v^2/c^2}$. Equation (28.7) was the beginning of the theory that mass depended on velocity.

Let's now go back to our calculation of the energy in the field, which led to Eq. (28.2). According to the theory of relativity, the energy U will have the mass U/c^2; Eq. (28.2) then says that the field of the electron should have the mass

$$m'_{\text{elec}} = \frac{U_{\text{elec}}}{c^2} = \frac{1}{2}\frac{e^2}{ac^2}, \tag{28.8}$$

which is not the same as the electromagnetic mass, m_{elec}, of Eq. (28.4). In fact, if we just combine Eqs. (28.2) and (28.4), we would write

$$U_{\text{elec}} = \frac{3}{4}m_{\text{elec}}c^2.$$

This formula was discovered before relativity, and when Einstein and others began to realize that it must always be that $U = mc^2$, there was great confusion.

28–4 The force of an electron on itself

The discrepancy between the two formulas for the electromagnetic mass is especially annoying, because we have carefully proved that the theory of electrodynamics is consistent with the principle of relativity. Yet the theory of relativity implies without question that the momentum must be the same as the energy times v/c^2. So we are in some kind of trouble; we must have made a mistake. We did not make an algebraic mistake in our calculations, but we have left something out.

In deriving our equations for energy and momentum, we assumed the conservation laws. We assumed that *all* forces were taken into account and that any work done and any momentum carried by other "nonelectrical" machinery was included. Now if we have a sphere of charge, the electrical forces are all repulsive and an electron would tend to fly apart. Because the system has unbalanced forces, we can get all kinds of errors in the laws relating energy and momentum. To get a *consistent* picture, we must imagine that something holds the electron together. The charges must be *held* to the sphere by some kind of rubber bands—something that keeps the charges from flying off. It was first pointed out by Poincaré that the rubber bands—or whatever it is that holds the electron together—must be included in the energy and momentum calculations. For this reason the extra nonelectrical forces are also known by the more elegant name "the Poincaré stresses." If the extra forces are included in the calculations, the masses obtained in two ways are changed (in a way that depends on the detailed assumptions). And the results are consistent with relativity; i.e., the mass that comes out from the momentum calculation is the same as the one that comes from the energy calculation. However, both of them contain *two* contributions: an electromagnetic mass and contribution from the Poincaré stresses. Only when the two are added together do we get a consistent theory.

It is therefore impossible to get all the mass to be electromagnetic in the way we hoped. It is not a legal theory if we have nothing but electrodynamics. Something else has to be added. Whatever you call them—"rubber bands," or "Poincaré stresses," or something else—there have to be other forces in nature to make a consistent theory of this kind.