The momentum in the field—the electromagnetic momentum—is proportional to v. It is just what we should have for a particle with the mass equal to the coefficient of v. We can, therefore, call this coefficient the *electromagnetic mass*, m_{elec}, and write it as

$$m_{\text{elec}} = \frac{2}{3}\frac{e^2}{ac^2}. \quad = \frac{4}{3}\text{ mass electrostatic field.} \tag{28.4}$$

28–3 Electromagnetic mass

Where does the mass come from? In our laws of mechanics we have supposed that every object "carries" a thing we call the mass—which also means that it "carries" a momentum proportional to its velocity. Now we discover that it is understandable that a charged particle carries a momentum proportional to its velocity. It might, in fact, be that the mass is just the effect of electrodynamics. The origin of mass has until now been unexplained. We have at last in the theory of electrodynamics a grand opportunity to understand something that we never understood before. It comes out of the blue—or rather, from Maxwell and Poynting—that any charged particle will have a momentum proportional to its velocity just from electromagnetic influences.

Let's be conservative and say, for a moment, that there are two kinds of mass—that the total momentum of an object could be the sum of a mechanical momentum and the electromagnetic momentum. The mechanical momentum is the "mechanical" mass, m_{mech}, times v. In experiments where we measure the mass of a particle by seeing how much momentum it has, or how it swings around in an orbit, we are measuring the total mass. We say generally that the momentum is the total mass ($m_{\text{mech}} + m_{\text{elec}}$) times the velocity. So the observed mass can consist of two pieces (or possibly more if we include other fields): a mechanical piece plus an electromagnetic piece. We know that there is definitely an electromagnetic piece, and we have a formula for it. And there is the thrilling possibility that the mechanical piece is not there at all—that the mass is all electromagnetic.

Let's see what size the electron must have if there is to be no mechanical mass. We can find out by setting the electromagnetic mass of Eq. (28.4) equal to the observed mass m_e of an electron. We find

$$\boxed{a = \frac{2}{3}\frac{e^2}{m_ec^2}.} \tag{28.5}$$

The quantity

$$\boxed{r_0 = \frac{e^2}{m_ec^2}} \tag{28.6}$$

is called the "classical electron radius"; it has the numerical value 2.82×10^{-13} cm, about one one-hundred-thousandth of the diameter of an atom.

Why is r_0 called the electron radius, rather than our a? Because we could equally well do the same calculation with other assumed distributions of charges—the charge might be spread uniformly through the volume of a sphere or it might be smeared out like a fuzzy ball. For any particular assumption the factor 2/3 would change to some other fraction. For instance, for a charge uniformly distributed throughout the volume of a sphere, the 2/3 gets replaced by 4/5. Rather than to argue over which distribution is correct, it was decided to define r_0 as the "nominal" radius. Then different theories could supply their pet coefficients.

Let's pursue our electromagnetic theory of mass. Our calculation was for $v \ll c$; what happens if we go to high velocities? Early attempts led to a certain amount of confusion, but Lorentz realized that the charged sphere would contract into a ellipsoid at high velocities and that the fields would change in accordance with the formulas (26.6) and (26.7) we derived for the relativistic case in Chapter 26. If you carry through the integrals for p in that case, you find that for an arbitrary velocity v, the momentum is altered by the factor $1/\sqrt{1 - v^2/c^2}$:

$$p = \frac{2}{3}\frac{e^2}{ac^2}\frac{v}{\sqrt{1 - v^2/c^2}}. \tag{28.7}$$