If we use the electronic charge $q_e$ for $q$ and the symbol $e^2$ for $q_e^2/4\pi\epsilon_0$, then

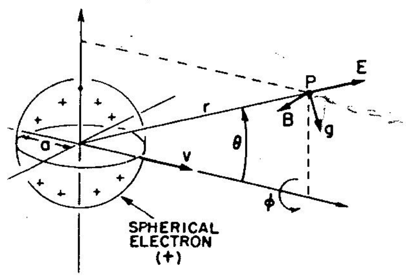
$$U_{\text{elec}} = \frac{1}{2}\frac{e^2}{a}. \quad = \frac{1}{8}\frac{Q_e^2}{\pi\epsilon_0 R_e} \tag{28.2}$$

It is all fine until we set $a$ equal to zero for a point charge—there's the great difficulty. Because the energy of the field varies inversely as the fourth power of the distance from the center, its volume integral is infinite. There is an infinite amount of energy in the field surrounding a point charge.

What's wrong with an infinite energy? If the energy can't get out, but must stay there forever, is there any real difficulty with an infinite energy? Of course, a quantity that comes out infinite may be annoying, but what really matters is only whether there are any *observable* physical effects. To answer that question, we must turn to something else besides the energy. Suppose we ask how the energy *changes* when we *move* the charge. Then, if the *changes* are infinite, we will be in trouble.

### 28–2 The field momentum of a moving charge

Suppose an electron is moving at a uniform velocity through space, assuming for a moment that the velocity is low compared with the speed of light. Associated with this moving electron there is a momentum—even if the electron had no mass before it was charged—because of the momentum in the electromagnetic field. We can show that the field momentum is in the direction of the velocity $\boldsymbol{v}$ of the charge and is, for small velocities, proportional to $v$. For a point $P$ at the distance $r$ from the center of the charge and at the angle $\theta$ with respect to the line of motion (see Fig. 28–1) the electric field is radial and, as we have seen, the magnetic field is $\boldsymbol{v} \times \boldsymbol{E}/c^2$. The momentum density, Eq. (27.21), is



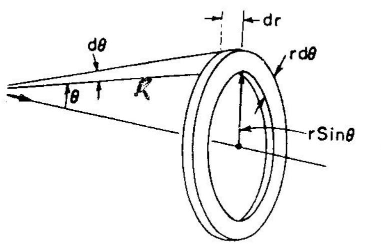

28–1. The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ and the ıtum density $\boldsymbol{g}$ for a positive elec- For a negative electron, $\boldsymbol{E}$ and $\boldsymbol{B}$ versed but $\boldsymbol{g}$ is not.

$$\boldsymbol{g} = \epsilon_0 \boldsymbol{E} \times \boldsymbol{B}.$$

It is directed obliquely toward the line of motion, as shown in the figure, and has the magnitude

$$g = \frac{\epsilon_0 v}{c^2} E^2 \sin\theta.$$

The fields are symmetric about the line of motion, so when we integrate over space, the transverse components will sum to zero, giving a resultant momentum parallel to $\boldsymbol{v}$. The component of $\boldsymbol{g}$ in this direction is $g \sin\theta$, which we must integrate over all space. We take as our volume element a ring with its plane perpendicular to $\boldsymbol{v}$, as shown in Fig. 28–2. Its volume is $2\pi r^2 \sin\theta\, d\theta\, dr$. The total momentum is then




28–2. The volume element n $\theta\, d\theta\, dr$ used for calculating the ɔmentum.

$$p = \int \frac{\epsilon_0 v}{c^2} E^2 \sin^2\theta\, 2\pi r^2 \sin\theta\, d\theta\, dr.$$

Since $E$ is independent of $\theta$ (for $v \ll c$), we can immediately integrate over $\theta$; the integral is

$$\int \sin^3\theta\, d\theta = -\int (1 - \cos^2\theta)\, d(\cos\theta) = -\cos\theta + \frac{\cos^3\theta}{3}.$$

The limits of $\theta$ are 0 and $\pi$, so the $\theta$-integral gives merely a factor of 4/3, and

$$p = \frac{8\pi}{3}\frac{\epsilon_0 v}{c^2}\int E^2 r^2\, dr.$$

The integral (for $v \ll c$) is the one we have just evaluated to find the energy; it is $q^2/16\pi^2\epsilon_0^2 a$, and

$$p = \frac{2}{3}\frac{q^2}{4\pi\epsilon_0}\frac{v}{ac^2},$$

or

$$p = \frac{2}{3}\frac{e^2}{ac^2} v. \quad \frac{2}{3}\cdot\frac{1}{4}\frac{Q_e^2}{\pi\epsilon_0 R_e} \tag{28.3}$$