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Electromagnetic Mass

28–1 The field energy of a point charge

In bringing together relativity and Maxwell's equations, we have finished our main work on the theory of electromagnetism. There are, of course, some details we have skipped over and one large area that we will be concerned with in the future—the interaction of electromagnetic fields with matter. But we want to stop for a moment to show you that this tremendous edifice, which is such a beautiful success in explaining so many phenomena, ultimately falls on its face. When you follow any of our physics too far, you find that it always gets into some kind of trouble. Now we want to discuss a serious trouble—the failure of the classical electromagnetic theory. You can appreciate that there is a failure of all classical physics because of the quantum-mechanical effects. Classical mechanics is a mathematically consistent theory; it just doesn't agree with experience. It is interesting, though, that the classical theory of electromagnetism is an unsatisfactory theory all by itself. There are difficulties associated with the *ideas* of Maxwell's theory which are not solved by and not directly associated with quantum mechanics. You may say, "Perhaps there's no use worrying about these difficulties. Since the quantum mechanics is going to change the laws of electrodynamics, we should wait to see what difficulties there are after the modification." However, when electromagnetism is joined to quantum mechanics, the difficulties remain. So it will not be a waste of our time now to look at what these difficulties are. Also, they are of great historical importance. Furthermore, you may get some feeling of accomplishment from being able to go far enough with the theory to see everything—including all of its troubles.

The difficulty we speak of is associated with the concepts of electromagnetic momentum and energy, when applied to the electron or any charged particle. The concepts of simple charged particles and the electromagnetic field are in some way inconsistent. To describe the difficulty, we begin by doing some exercises with our energy and momentum concepts.

First, we compute the energy of a charged particle. Suppose we take a simple model of an electron in which all of its charge q is uniformly distributed on the surface of a sphere of radius a, which we may take to be zero for the special case of a point charge. Now let's calculate the energy in the electromagnetic field. If the charge is standing still, there is no magnetic field, and the energy per unit volume is proportional to the square of the electric field. The magnitude of the electric field is $q/4\pi\epsilon_0 r^2$, and the energy density is

$$u = \frac{\epsilon_0}{2}E^2 = \frac{q^2}{32\pi^2\epsilon_0 r^4}.$$

To get the total energy, we must integrate this density over all space. Using the volume element $4\pi r^2\,dr$, the total energy, which we will call U_{elec}, is

$$U_{\text{elec}} = \int \frac{q^2}{8\pi\epsilon_0 r^2}\,dr.$$

This is readily integrated. The lower limit is a, and the upper limit is ∞, so

$$U_{\text{elec}} = \frac{1}{2}\frac{q^2}{4\pi\epsilon_0}\frac{1}{a}. \tag{28.1}$$